ON THE SOLUTION OF THE TWO-POINT BOUNDARY VALUE PROBLEM IN THE THEORY OF CONTINUOUS-FLOW REACTORS

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An asymptotic expansion is derived for the solution of two-point boundary value problem of a uniquely perturbed system of two equations with small parameters at higher derivatives, a problem that arises in the theory of continuous-flow reactors. Formulas are obtained for concentration of reagents, which are uniformly valid throughout a reactor with two-stage reaction sequence and accurate within the quadratic term in small parameters. A comparison is made of the numerical and asymptotic solutions.

It would seem that the investigation presented in [2] of the steady operation mode of a reactor with single-stage isothermal reaction at high Péclet numbers, based on approximate methods of nonlinear mechanics, was the first to tackle this subject. Formal extension of the method used in [2] to a system of equations was carried out in [3,4] for certain limit cases.

1. The complete system of equations defining an isothermal reactor with two-stage reaction sequence includes equations of conservation for the initial and intermediate products [5]. A number of assumptions was used in [5,6] for the mathematical formulation of the steady operation mode of reactor. It is as follows:

$$\varepsilon_\alpha y_\alpha'' = y_\alpha' + \lambda_\alpha y_\alpha^n, \quad \varepsilon_\beta y_\beta'' = y_\beta' + \lambda_\beta y_\beta^m - \lambda_\alpha y_\alpha^n$$

$$\varepsilon_\alpha y_\alpha' = y_\alpha - 1, \quad \varepsilon_\beta y_\beta' = y_\beta \quad \text{for } t = +0 \tag{1.1}$$

$$y_\alpha' = y_\beta' = 0 \quad \text{for } t = 1$$

$$(\varepsilon_\gamma = D_\gamma / u_0 L, \quad \lambda_\gamma = k_\gamma L y_\alpha^{s-1}(-0) / u_0; \quad \gamma = \alpha, \beta; s = n, m)$$

where primes denote differentiation with respect to arguments of functions; ε_γ^{-1} is the Péclet number with $\gamma = \alpha$ or β denoting the initial or intermediate products, respectively, u_0 is the mean velocity of mixture in the reactor duct; D_γ is the coefficient of partial diffusion; L is the reactor length; $t = z / L$ is the dimensionless coordinate along the reactor axis; k_γ is the constant of the reaction rate; $y_\gamma = c_\gamma / c(-0)$ is the dimensionless partial concentration, where $c(-0)$ is the partial concentration at the reactor inlet. The reactor occupies the region $0 \leqslant t \leqslant 1$.

We investigate the most interesting from the practical point of view case of high Péclet numbers, i.e. $\varepsilon_\gamma \ll 1$, in which (1.1) represents a two-point boundary value problem for a system of two equations with small parameters at higher derivatives. Such system is uniquely perturbed with respect to the small parameters ε_α and ε_β.

Let us derive the solution of problem (1.1) in the outer region which is represented by the half-interval of $0 \leqslant t < 1$. We represent the initial product concentration in the form of the asymptotic expansion

$$y_\alpha(t) = \chi_\alpha(t) = \sum_{n=0}^{\infty} \varepsilon_\alpha{}^n \chi_\alpha^{(n)}(t) \qquad (1.2)$$

in the small parameter ε_α. Owing to the relative complexity of the second equation of system (1.1) we represent the intermediate product by the asymptotic series

$$y_\beta(t) = \chi_\beta(t) = \sum_{k=n=0}^{\infty} \varepsilon_\alpha{}^k \varepsilon_\beta{}^n \chi_\beta^{(k,\,n)}(t) \qquad (1.3)$$

in two small parameters ε_α and ε_β.

The derivation of asymptotic expansions of solutions for (1.1) and (1.2) is deter-mined by the system of zero approximation which is obtained from (1.1) by setting $\varepsilon_\alpha = \varepsilon_\beta = 0$ and defines the work of an isothermal reactor with a consecutive re-action in a perfect displacement mode. The concentration of reagents at the reactor inlet is used as the boundary condition. In the mode of perfect displacement the value of the initial product along the reactor length is determined by the simple formula

$$\chi_\alpha^{(0)}(t) = \{-(1-n)\lambda_\alpha t + 1\}^{-1} \qquad (1.4)$$

The analytic solution of the second equation of the zero approximation which de-termines the value of the intermediate product is generally difficult. Let us consider the second order reaction, i.e. $n = m = 2$. In that case the sought equation is a Riccati equation, and the intermediate product concentration is determined by the fol-lowing formulas:

$$\chi_\beta^{(0,0)}(t) = A\,\chi_\alpha^{(0)}(t) + z(t) \qquad (1.5)$$

$$z(t) = -(\chi_\alpha^{(0)}(t))^a \{1 + A[\lambda_\beta/(\lambda_\alpha a)][(\chi_\alpha^{(0)}(t))^a - 1]\}^{-1}$$

$$A = (\lambda_\alpha/(2\lambda_\beta))(1+a),\ a = (1 + 4\lambda_\beta/\lambda_\alpha)^{1/2}$$

In the asymptotic expansion of solution in the outer region the effects of diffusion on the processes that take place in the reactor is taken into account in subsequent approximations with respect to the small parameters ε_α and ε_β. It follows from (1.1) that the first and subsequent terms of outer expansions (1.2) and (1.3) conform to the nonhomoge-neous linear equations

$$\chi_\alpha^{(m)'} + 2\lambda_\alpha \chi_\alpha^{(0)} \chi_\alpha^{(m)} = F_\alpha^{(m)}(t), \quad m \geqslant 1 \qquad (1.6)$$

$$\chi_\beta^{(m,\,s)'} + 2\lambda_\beta \chi_\beta^{(0,\,0)} \chi_\beta^{(m,\,s)} = F_\beta^{(m,\,s)}(t), \quad m+s \geqslant 1$$

$$\chi_\alpha^{(m)} = \chi_\alpha^{(m-1)}, \quad \chi_\beta^{(m,\,s)} = \chi_\beta^{(m,\,s-1)} \quad \text{for } t = +0$$

The right-hand sides of equations of this system are determined by lower approxi-mation solutions using the following formulas:

$$F_\alpha^{(m)} = \chi_\alpha^{(m-1)''} - \lambda_\alpha \sum_{1 \leqslant n \leqslant m-1} \chi_\alpha^{(m-n)} \chi_\alpha^{(n)}$$

$$F_\beta^{(m,\,n)} = \chi_\beta^{(m,\,s-1)'} - \lambda_\beta \sum_{1\leqslant n\leqslant m-1} \sum_{1\leqslant k\leqslant s-1} \chi_\beta^{(m-n,\,s-k)} \chi_\beta^{(n,\,k)} +$$

$$\lambda_\alpha \delta_{s0} \sum_{0\leqslant n\leqslant m} \chi_\alpha^{(m-n)} \chi_\alpha^{(n)}$$

Solutions of the first and subsequent approximations may be expressed in terms of the zero approximation, which simplifies the derivation of the asymptotic expansion in the outer region in comparison with the method used in [2]. For instance, the first and second terms of the asymptotic expansion (1.2) for the initial product can be represented by formulas

$$\chi_\alpha^{(1)}(t) = -\chi_\alpha^{(0)2}(t)\,[1 + 2\ln\chi_\alpha^{(0)}(t)]$$

$$\chi_\alpha^{(2)}(t) = (\lambda_\alpha\chi_\alpha^{(0)}(t))^2\,\{9\,(\chi_\alpha^{(0)}(t) - 1) + 4\,[\chi_\alpha^{(0)}(t)\,\times$$

$$\ln\chi_\alpha^{(0)}(t)\,(2 + \ln\chi_\alpha^{(0)}(t)) + 1]\}$$

and, owing to the linearity of system (1.6) with respect to the unknown functions, its solution can be determined by the more general formulas

$$\chi_\alpha^{(m,\,s)}(t) = \chi_\alpha^{(0)2}(t)\left\{\chi_\alpha^{(m-1)'}(+0) + \right. \tag{1.7}$$

$$\left. \int_0^t dt'\,(\chi_\alpha^{(0)}(t'))^{-2}\,F_\alpha^{(m)}(t')\right\} + O\,(\varepsilon_\alpha)$$

$$\chi_\beta^{(m,\,s)}(t) = \varphi(t)\left\{\chi_\beta^{(m,\,s-1)'}(+0) + \int_0^t dt'\,F_\beta^{(m,\,s)}(t')\,\varphi^{-1}(t') + \right.$$

$$\left. O\,(\max\,(\varepsilon_\alpha,\,\varepsilon_\beta))\right\}$$

$$\varphi(t) = \exp\left\{-2\lambda_\beta\int_0^t \chi_\beta^{(0,\,0)}(t')\,dt'\right\}$$

The first of these is obtained with an accuracy within $O\,(\varepsilon_\alpha)$ and the second within $O\,(\max\,(\varepsilon_\alpha,\,\varepsilon_\beta))$.

Functions $\chi_\beta^{(n,m)}(t)$ are shown in Fig. 1 for $\lambda_\alpha = \lambda_\beta = 1$ and $\varepsilon_\alpha = \varepsilon_\beta = 0.1$, and the values of n and m are shown there in parentheses.

2. To have the solution-which is to be uniformly valid in the whole segment $0 \leqslant t \leqslant 1$-conform to boundary conditions we introduce at the reactor outlet in the neighborhood of point $t = 1$ a "boundary layer". Substituting the variables $\eta_\gamma = (1 - t)\,/\,\varepsilon_\gamma$ $(\gamma = \alpha,\,\beta)$ for the input system of Eqs. (1.1) in the "boundary layer", in the particular case of second order reaction we obtain

$$\psi_\alpha'' + \psi_\alpha' = \varepsilon_\alpha\lambda_\alpha\psi_\alpha^2, \qquad \psi_\beta'' + \psi_\beta' = \varepsilon_\beta\,[\lambda_\beta\psi_\beta^2 - \lambda_\alpha\psi_\alpha^2] \tag{2.1}$$

$$\psi_\alpha' = \psi_\beta' = 0 \quad \text{for } \eta_\gamma = 0$$

$$(y_\alpha(t) = \psi_\alpha(\eta_\alpha), \qquad y_\beta(t) = \psi_\beta(\eta_\beta))$$

We define the asymptotic expansion of solution of system (2.1) in small parameters ε_α and ε_β by the formulas

$$\psi_\alpha(\eta_\alpha) = \sum_{n=0}^{\infty} \varepsilon_\alpha{}^n \psi_\alpha^{(n)}(\eta_\alpha), \quad \psi_\beta(\eta_\beta) = \sum_{m,\,n=0}^{\infty} \varepsilon_\beta{}^m \varepsilon_\beta{}^n \psi_\beta^{(m,\,n)}(\eta_\beta) \qquad (2.2)$$

The perfect displacement mode is not realized in the inner region, since the diffusion transport of the reagent is there comparable to convective transport. Concentration changes due to chemical reaction are neglected. The system of zero approximation equations is obtained from (2.1) by setting
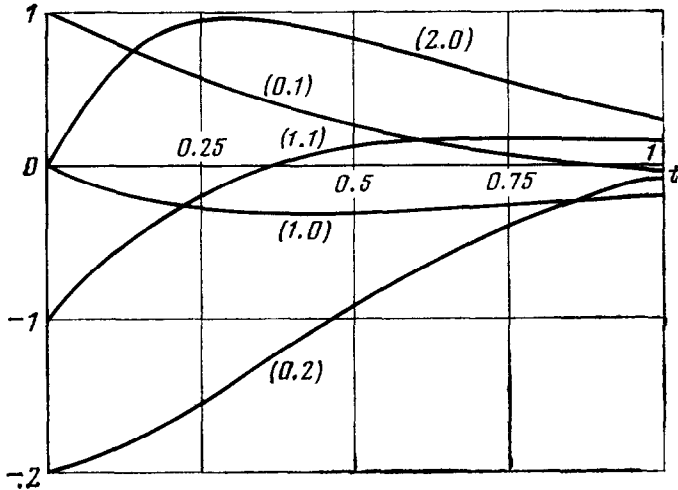

Fig. 1

$\varepsilon_\alpha = \varepsilon_\beta = 0$, and has the simple solution $\psi_\alpha^{(0)}(\eta_\alpha) = h_{\alpha0}^{(0)}$, $\psi_\beta^{(0,0)}(\eta_\beta) = h_\beta^{(0,0)}$. The constants $h_{\alpha0}^{(0)}$ and $h_{\beta0}^{(0,0)}$ which appear in the zero approximation are determined in the course of joining the outer solution, represented by the asymptotic expansion (1.2), (1.3), with solution (2.2) for the boundary layer. Changes of the reagent concentration due to the chemical reaction are taken into account in the first and subsequent approximations of expansion (2.2).

From the system of first approximation equations

$$\psi_\alpha^{(n)''} + \psi_\alpha^{(n)'} = F_\alpha^{(n)}, \quad n=1; \quad \psi_\beta^{(m,\,n)''} + \psi_\beta^{(m,\,n)'} = F_\beta^{(m,\,n)}, \quad m+n=1 \qquad (2.3)$$

$$\psi_\alpha^{(n)'} = \psi_\beta^{(m,\,n)'} = 0 \quad \text{for} \quad \eta_\alpha = \eta_\beta = 0$$

$$F_\alpha^{(n)} = \lambda_\alpha \psi_\alpha^{(0)2}, \quad F_\beta^{(0,1)} = \lambda_\beta \psi_\beta^{(0,0)2} - \lambda_\alpha \psi_\alpha^{(0)2}, \quad F_\beta^{(1,0)} = 0$$

we obtain for the first term of expansion (2.2) the expression

$$\psi_\alpha^{(1)} = h_{\alpha0}^{(1)} + h_\alpha \varphi_\alpha^{(1)}, \quad \psi_\beta^{(0,1)} = h_{\beta0}^{(0,1)} + h_\beta^{(0,1)} \varphi_\beta \qquad (2.4)$$

$$\psi_\beta^{(1,0)} = h_{\beta0}^{(1,0)}, \quad h_\alpha^{(1)} = \lambda_\alpha h_\alpha^{(0)2}, \quad h_\beta^{(0,1)} = \lambda_\beta h_\beta^{(0,0)2} - \lambda_\alpha h_\alpha^{(0)2}$$

$$\varphi_\gamma = \eta_\gamma + e^{-\eta_\gamma}, \quad \gamma = \alpha, \beta$$

which satisfies the boundary condition that the derivative must vanish at the right-hand end of the segment. In that case the constants $h_\alpha^{(1)}$ and $h_\beta^{(0,1)}$ are functions of $h_\alpha^{(0)}$ and $h_\beta^{(0,0)}$, while constants $h_{\alpha0}^{(1)}, h_{\beta0}^{(0,1)}, h_{\beta0}^{(1,0)}$ which appear in (2.4)

are determined in the course of joining the sought solution with solution (1.1),(1.2).

The described above algorithm of solution derivation is used for calculating subsequent expansions of (2.2). From the second approximation equations which differ from Eq. (2.3) by the value of functions

$$F_\alpha^{(2)} = 2\lambda_\alpha h_{\alpha 0}^{(0)}\psi_\alpha^{(1)}, \quad F_\beta^{(1,1)} = (F_\beta^{(0,0)} - F_\alpha^{(2)}),$$
$$F_\beta^{(0,2)} = 2\lambda_\beta h_{\beta 0}^{(0,0)}\psi_\beta^{(0,1)}, \quad F_\beta^{(2,0)} = 0$$

in its right-hand side and are determined by solutions of lower approximations, we thus obtain the second term of expansion (2.2)

$$\psi_\alpha^{(2)} = (F_\alpha^{(2)}\varphi_\alpha + h_\alpha h_\alpha^{(1)}\Phi_\alpha + h_{\alpha 0}^{(2)}) \tag{2.5}$$

$$\psi_\beta^{(0,2)} = F_\beta^{(0,2)}\varphi_\beta + h_\beta h_\beta^{(0,1)}\Phi_\beta + h_{\beta 0}^{(0,2)}$$

$$\psi_\beta^{(2,0)} = h_{\beta 0}^{(2,0)}, \quad \psi_\beta^{(1,1)} = (F_\beta^{(0,2)} - F_\alpha^{(2)})\varphi_\beta -$$
$$h_\alpha h_\alpha^{(1)}\left[\frac{\varepsilon_\alpha}{\varepsilon_\beta}\left(\frac{\eta_\beta^2}{2} - \eta_\beta\right) + \Delta_\beta^{-1}e^{-\eta_\alpha}\right]$$

$$h_\alpha = 2\lambda_\alpha\psi_\alpha^{(0)}, \quad h_\beta = 2\lambda_\beta\psi_\beta^{(0,0)}, \quad \Delta_\beta = \frac{\varepsilon_\beta}{\varepsilon_\alpha}\left(\frac{\varepsilon_\beta}{\varepsilon_\alpha} - 1\right)$$

$$\Phi_\gamma = \left(\frac{\eta_\gamma^2}{2} - \eta_\gamma\right) - (\eta_\gamma + 1)e^{-\eta_\gamma}, \quad \gamma = \alpha, \beta$$

which satisfies the boundary condition that the derivative must vanish at the reactor outlet. The expression for $\psi_\beta^{(1,1)}$ was obtained on the assumption that $|\varepsilon_\alpha - \varepsilon_\beta| = O(\varepsilon_\alpha + \varepsilon_\beta)$. When $|\varepsilon_\alpha - \varepsilon_\beta| = o(\varepsilon_\alpha + \varepsilon_\beta)$ it is necessary to substitute in $\psi_\beta^{(1,1)}$ the quantity $-(\eta_\beta + 1)$ for Δ_β^{-1}. Constants $h_{\alpha 0}^{(2)}$ and $h_\beta^{(m,k)}$ $(m + k = 2)$ are determined, as in lower approximations, in the course of joining asymptotic expansions. The general term of the asymptotic expansion (2.2) can be expressed in terms of exponents of quantities $\psi_\alpha^{(n)} = O(\varepsilon_\alpha^n \exp \cdot \{-\eta_\alpha\})$ and $\psi_\beta^{(n,m)} = O(\max(\varepsilon_\alpha, \varepsilon_\beta)^{n+m} \exp\{-\eta_\beta\})$ for the initial and intermediate products, respectively.

3. The constants which appear in solutions of the boundary layer are obtained in the course of joining the asymptotic expansions of solutions in the inner and outer regions. For this we pass to "intermediate variables" $\eta_\gamma = \eta_\gamma^* / \mu_\gamma$ and $t = 1 - (\varepsilon_\gamma / \mu_\gamma)\eta_\gamma^*$ $(\gamma = \alpha, \beta)$ on the assumption that $\mu_\gamma = \mu_\gamma(\varepsilon_\gamma)$ and $\mu_\gamma = o(\varepsilon_\gamma)$. When joining the asymptotic expansions it is convenient to represent the outer solution, which is a function of $t = 1 - (\varepsilon_\gamma / \mu_\gamma)\eta_\gamma^*$, in the form of a Taylor series in the small parameter $\varepsilon_\gamma / \mu_\gamma$. We restrict the joining of the inner and outer asymptotic expansions to the zero, first, and second powers of the intermediate variable, which corresponds to joining to within terms of order $(\varepsilon_\gamma / \mu_\gamma)^2 \ll 1$. Thus in the zero approximation for the outer solution in small parameters ε_α and ε_β, neglecting small exponential terms in the inner solution, and equating coefficients at corresponding powers of the intermediate variable of the inner and outer expansions, we obtain the system

$$\chi_\alpha^{(0)}(1) = h_{\alpha 0}^{(0)}, \quad \chi_\beta^{(0,0)}(1) = h_{\beta 0}^{(0,0)}, \quad \chi_\alpha^{(0)'}(1) = -h_\alpha^{(1)}, \quad \chi_\beta^{(0,0)'}(1) = h_\beta^{(0,1)} \tag{3.1}$$
$$\chi_\alpha^{(0)''} = 2\lambda_\alpha h_\alpha^{(1)}, \quad \chi_\beta^{(0,0)''}(1) = 2[\lambda_\beta h_\beta^{(0,1)} - \lambda_\alpha h_\alpha^{(1)}]$$

The first two formulas directly determine constants $h_{\alpha 0}{}^{(0)} = \chi_\alpha{}^{(0)}$ (1) and $h_{\beta 0}{}^{(0,0)} = \chi_\beta{}^{(0,0)}$ (1) in terms of the zero term of the outer solution asymptotic expansion, while the fourth, fifth and sixth formulas are identically satisfied.

To prove the last statement it is sufficient to express the first and second derivatives of $\chi_\alpha^{(0)}$ and $\chi_\beta^{(0,0)}$ in terms of these functions, using the system of equations for the determination of the zero term of the outer solution asymptotic expansion and, then, substitute these into (3.1). To remain within the required accuracy of matching the zero approximation solutions with respect to parameters ε_α and ε_β we restrict the joining of the outer and inner solutions at zero and first powers of the intermediate variable. As previously, disregarding exponentially small terms in the inner solution asymptotic expansion and equating coefficients at like powers of the intermediate variable of the inner and outer expansions, we obtain a system of six equations. Three of these equations determine the constants

$$h_{\alpha 0}^{(1)} = \chi_\alpha^{(1)} \ (1), \ h_{\beta 0}^{(0,1)} = \chi_\beta^{(0,1)} \ (1), \quad \text{and} \quad h_{\beta 0}^{(1,0)} = \chi_\beta^{(1,0)} \ (1)$$

in terms of first terms of the outer solution asymptotic expansion. The remaining equations of that system represent identities. This can be proved as above in the example of the algorithm for defining the unknown constants in the inner solution in the case of zero approximation with respect to the small parameters ε_α and ε_β.

It should be pointed out that in the theory of the isothermal reactor with single stage reaction sequence [2] the inner solution was obtained by the iteration method. Owing to some confusion as regards the orders with respect to small parameter of the considered there problem, the determination of the unknown constants of the inner solution became somewhat complicated. In more complex cases of systems with several small parameters that method of deriving asymptotic solution was found to be ineffective, since it made it impossible to analytically determine the unknown constants and to obtain a solution that is uniformly applicable along the whole segment. On the other hand, the described algorithm for the derivation of asymptotic expansion of the two-point boundary value problem makes it possible to obtain a uniformly valid solution for the more complex system of equations with several small parameters.

In the case of an isothermal reactor with a two-stage reaction of the second order we separate the common part, and obtain the interval of the asymptotic series of solution of problem (1.1) which is uniformly valid throughout the interval

$$y_\alpha^{(n=2)}(t) = \sum_{n=0}^{n=2} \varepsilon_\alpha^n \left\{ \chi_\alpha^{(n)}(t) - \sum_{l=0}^{l+n=2} \frac{1}{l!} \frac{\partial^l \chi_\alpha^{(n)}(1)}{\partial t^l} (1-t)^l + \right. \tag{3.2}$$

$$\left. \psi_\alpha^{(n)}\left(\frac{1-t}{\varepsilon_\alpha}\right) \right\}, \quad y_\beta^{(n=2)}(t) = \sum_{k,\,m=0}^{n=k+m=2} \varepsilon_\alpha^k \varepsilon_\beta^m \left\{ \chi_\beta^{(k,\,m)}(t) - \sum_{l=0}^{l+n=2} \frac{1}{l!} \times \right.$$

$$\left. \frac{\partial^l \chi_\beta^{(n,\,m)}(1)}{\partial t^l} (1+t)^l + \psi_\beta^{(k,\,m)}\left(\frac{1-t}{\varepsilon_\beta}\right) \right\}$$

which is accurate to quadratic terms in small parameters ε_α and ε_β. The last two terms determine the exponential part of solution which is real in the boundary layer near $t = 1$. Expressions for the terms of asymptotic expansion in the outer and inner regions $\chi_\alpha^{(n)}$, $\chi_\beta^{(n,m)}$ and $\psi_\alpha^{(n)}$, $\psi_\beta^{(n,m)}$, respectively, appear in (1.4), (1.5), (1.7), (2.4), and (2.5). Because of the requirement for the derivative of the reagent

concentration to vanish at the reactor outlet, stipulated by the boundary condition, the effect of the exponential part of solution (3.2) makes itself felt already in the first approximation with respect to the small parameters ε_α and ε_β. In that approximation this effect on the initial and intermediate products is determined by partial diffusion of the respective product. However in the second approximation the boundary layer for the intermediate product is formed under the effect of diffusion of the intermediate, as well as of the initial product.

Table 1

	t	$y_\alpha \times 10^3$	$v_\alpha^{(1)} \times 10^3$	$v_\alpha^{(2)} \times 10^3$	$v_\beta \times 10^3$	$v_\beta^{(1)} \times 10^3$	$v_\beta^{(2)} \times 10^3$
A	0.0	924	900	940	72	100	60
	0.2	797	789	799	194	204	191
	0.4	698	698	697	273	279	277
	0.6	620	623	618	333	331	335
	0.8	560	564	558	369	364	370
	1.0	527	535	527	385	378	384
B	0.0	878	800	960	065	100	040
	0.2	769	746	784	173	174	179
	0.4	684	683	678	244	231	257
	0.6	616	627	605	288	272	299
	0.8	567	585	556	314	299	320
	1.0	545	570	540	326	313	338
C	0.0	813	600	240	105	200	040
	0.2	658	648	634	269	235	349
	0.4	550	579	487	366	322	431
	0.6	472	508	418	422	388	456
	0.8	418	455	379	452	429	465
	1.0	394	431	374	464	452	474

Let us turn to the solution of the two-point boundary value problem (1.1)   which represents an infinite segment of an asymptotic series. It is possible to maintain that for a finite series of asymptotic expansion there exists a positive number $\varepsilon > 0$ and a number N dependent on ε such that for all $\varepsilon_\alpha, \varepsilon_\beta < \varepsilon$ and $n > N$ the remainder term $|y_\gamma^{(n)} - y_\gamma| < O(\varepsilon^{n+1})$, $\gamma = \alpha, \beta$. The asymptotic solution y_γ $(\gamma = \alpha, \beta)$ of problem (1.1) then exists and is unique. To prove the above statements it is sufficient to take into consideration the estimate of the common term of asypmtotic expansions of the inner and outer solutions which appear at the ends of Sections 1 and 2, and use the standard method of proof proposed in [3]. The problem of specific determination of the quantity ε which represents the radius of convergence of asymptotic expansions, remains however open.

The concentration of initial and intermediate products along the reactor is shown for comparison in Table 1 for the following cases : A $\varepsilon_\alpha = \varepsilon_\beta = 0.1$, $\lambda_\alpha = \lambda_\beta = 1$, B $\varepsilon_\alpha = 0.2$, $\varepsilon_\beta = 0.1$, $\lambda_\alpha = 1$, $\lambda_\beta = 2$, and C $\varepsilon_\alpha = 0.2$, $\varepsilon_\beta = 0.1$, $\lambda_\alpha = 2$, $\lambda_\beta = 1$. The tabulated data were obtained on a computer by the trial-and-error method using solutions of (1.1) and the value of the finite series of asymptotic expansion of the two-point boundary value problem solution with two and three terms for various sets of parameters. The tabulated data shows that the three-term asymptotic expansion satisfactorily converges to the numerical solution in a fairly wide range of parameters that are of practical

interest. The convergence improves with decreasing parameters ε_γ, λ_γ and $\gamma = \alpha$, β. A worsening of convergence is noticeable in case C when $\varepsilon_\alpha = 0.2$, $\varepsilon_\beta = 0.1$, $\lambda_\alpha = 2$, $\lambda_\beta = 1$ at the reactor inlet.

The paper [7] should be noted in this connection, where the criterion of asymptotic solution convergence was obtained by matching solutions of the two-point problems in the inner and outer coordinates at the reactor inlet. However such approach to the derivation of convergence criterion is intuitive, and the criterion itself is unnecessarily stringent.

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